

High energy particle collisions and geometry of horizon

O. B. Zaslavskii

Department of Physics and Technology,

Kharkov V.N. Karazin National University,

4 Svoboda Square, Kharkov 61022, Ukraine and

Institute of Mathematics and Mechanics, Kazan Federal University,

*18 Kremlyovskaya St., Kazan 420008, Russia**

We consider collision of two geodesic particles near the lightlike surface (black hole horizon or naked singularity) of such an axially symmetric rotating or static metric that the coefficient $g_{\phi\phi} \rightarrow 0$ on this surface. It is shown that the energy in the centre of mass frame $E_{c.m.}$ is indefinitely large even without fine-tuning of particles' parameters. Kinematically, this is collision between two rapid particles that approach the horizon almost with the speed of light but at different angles (or they align along the normal to the horizon too slowly). The latter is the reason why the relative velocity tends to that of light, hence to high $E_{c.m.}$. Our approach is model-independent. It relies on general properties of geometry and is insensitive to the details of material source that supports the geometries of the type under consideration. For several particular models (the stringy black hole, the Brans-Dicke analogue of the Schwarzschild metric and the Janis-Newman-Winicour one) we recover the results found in literature previously.

PACS numbers: 04.70.Bw, 97.60.Lf

I. INTRODUCTION

Several years ago, an interesting observation was made by Bañados, Silk and West (the BSW effect, after the names of its authors). It turned out that if two particles collide near the black hole horizon, their energy in the centre of mass frame $E_{c.m.}$ can grow unbounded [1]. At

*Electronic address: zaslav@ukr.net

first, this was obtained for the Kerr black hole but later on, it was shown that this is a generic feature of rotating black holes [2]. Typically, $E_{c.m.}$ remains modest. It grows unbounded under special conditions only. Namely, one of particles should have special relation between the energy and the angular momentum (so-called critical particle), whereas another one should be not fine-tuned (so-called usual) - see aforementioned papers for details.

Meanwhile, several papers appeared in which it was found that unbounded $E_{c.m.}$ can be obtained without fine-tuning at all! As there is a sharp contrast (or even seeming contradiction) between these results and the standard picture described in the first paragraph, special explanation is needed here. In [3], unbounded $E_{c.m.}$ were obtained for the naked singularities described by the Janis-Newman-Winicour metric. In [4], this was obtained for stringy black holes and in [5] for the Brans-Dicke analogue of the Kerr black hole (so-called BDK metric [6]). Thus very different metrics and completely different types of spacetime (naked singularities and black holes) give the same results and this appeals to explanation.

In the present paper, we consider "dirty" black holes (see, e.g. [7]) and develop a general approach that handles such cases independently of the details of a metric and material source that supports it. This approach agrees with previous particular results. The type of spacetime under consideration extends the set of geometries for which the high energy collisions are possible.

Throughout the paper, we put fundamental constants $G = c = 1$.

II. METRIC, EQUATIONS OF MOTION

We consider the general metric of the form

$$ds^2 = -N^2 dt^2 + g_\phi (d\phi - \Omega dt)^2 + \frac{dr^2}{A} + g_\theta d\theta^2 \quad (1)$$

in which all metric coefficients do not depend on t and ϕ . Therefore, the energy $E = -mu_0$ and the angular momentum $L = mu_\phi$ are conserved, where m is the particle's mass, $u^\mu = \frac{dx^\mu}{d\tau}$ is the four-velocity, τ is the proper time. In what follows, we consider motion within the equatorial plane $\theta = \frac{\pi}{2}$ only. Then, we redefine the radial coordinate in such a way that $A = N^2$. The equations of motion for geodesics read

$$m\dot{t} = \frac{X}{N^2}, \quad (2)$$

$$X = E - \Omega L, \quad (3)$$

$$m\dot{\phi} = \frac{L}{g_\phi} + \frac{\Omega X}{N^2}, \quad (4)$$

$$m\dot{r} = \sigma Z, \quad (5)$$

where $\sigma = \pm 1$ depending on the direction of radial motion,

$$Z = \sqrt{X^2 - N^2(m^2 + \frac{L^2}{g_\phi})}. \quad (6)$$

III. GENERAL FORMULAS FOR COLLISION

Let two particles 1 and 2 collide. The energy in their centre of mass frame is equal to

$$E_{c.m.}^2 = -(m_1 u_{1\mu} + m_2 u_{2\mu})(m_1 u_1^\mu + m_2 u_2^\mu) = m_1^2 + m_2^2 + 2m_1 m_2 \gamma, \quad (7)$$

where

$$\gamma = -u_{1\mu} u_2^\mu \quad (8)$$

is the Lorentz gamma factor of relative motion.

We assume that in (5) $\sigma_1 = \sigma_2 = -1$ that is typical of particle motion near black holes. Then, it follows from (2) - (6) that

$$m_1 m_2 \gamma = \frac{X_1 X_2 - Z_1 Z_2}{N^2} - \frac{L_1 L_2}{g_\phi}. \quad (9)$$

IV. NEAR-HORIZON COLLISIONS OF USUAL PARTICLES

If the black hole horizon exists, it is located at $N = 0$. We include into consideration also singularities for which $N = 0$. Although, strictly speaking, such a surface cannot be called a horizon, we will use for brevity the term "horizon" both in the regular and singular cases. According to [1] and its generalization in [2], if collision occurs at the point with small N , the factor γ can be indefinitely large, provided one of particles is fine-tuned ("critical") whereas another one ("usual") is not. In doing so, it was tacitly assumed that g_ϕ remains nonzero.

But let us consider another case, when simultaneously $N \rightarrow 0$ and

$$g_\phi \rightarrow 0. \quad (10)$$

We will be interested in collision of usual (not fine-tuned) particles. We denote $\frac{N^2}{g_\phi} = b^2$. Then, for collision at the point with small N , we have

$$m_1 m_2 \gamma \approx \frac{D}{N^2}, \quad (11)$$

$$D = X_1 X_2 - \sqrt{X_1^2 - L_1^2 b^2} \sqrt{X_2^2 - L_2^2 b^2} - b^2 L_1 L_2. \quad (12)$$

It can be also rewritten in the form

$$m_1 m_2 \gamma \approx \frac{C}{g_\phi}, \quad C = \frac{D}{b^2}, \quad (13)$$

useful for small b (see below).

Introducing α_i ($i = 1, 2$) according to

$$b L_i \equiv X_i \sin \alpha_i, \quad -\frac{\pi}{2} \leq \alpha_i \leq \frac{\pi}{2}, \quad (14)$$

we can rewrite (11) as

$$m_1 m_2 \gamma \approx \frac{X_1 X_2 d}{N^2}, \quad (15)$$

$$d = 1 - \cos(\alpha_1 - \alpha_2). \quad (16)$$

Now, we consider two different limiting cases depending on the near-horizon behavior of b .

A. $b \rightarrow 0$

Then, $D = O(b^2)$ but C is, in general, nonzero,

$$C = \frac{(L_1 X_2 - L_2 X_1)^2}{2 X_1 X_2}. \quad (17)$$

If g_ϕ is finite on the horizon, the quantity γ is finite as well according to (13). This is in accord with the fact that collision of two usual particles cannot produce unbounded $E_{c.m.}$ [2]. However, for extremely small g_ϕ , this rule is not valid anymore and, according to (13), we obtain indefinitely large γ .

B. $b \rightarrow \infty$

For nonzero $L_{1,2}$ this is impossible since it is seen from (6) that this violates the condition $Z^2 > 0$. However, we can adjust L_1 and L_2 to the big b at the point of collision taking small values of L_i to keep the right hand side of (14) finite. Then, eqs. (15), (16) retain their validity. Now, we can neglect L_i in (3), so $X_i \approx E_i$.

C. Collision in the turning point

Let near-horizon collision occur in the turning point (say, for particle 1). Then, $Z_1 = 0$. Taking for definiteness $L_1 > 0$ we have $\alpha_1 = \frac{\pi}{2}$. $d = 1 - \sin \alpha_2$.

Let this be the turning point for particle 2 as well. Then, if $L_2 > 0$, $\alpha_2 = \frac{\pi}{2}$, $d = 0$ in eq. (16), the effect of unbounded γ is absent. If $L_2 < 0$, $\alpha_2 = -\frac{\pi}{2}$ and $d = 2$,

$$m_1 m_2 \gamma \approx \frac{2E_1 E_2}{N^2}. \quad (18)$$

V. KINEMATIC EXPLANATION

It is of interest to elucidate the underlying reason that gives rise to unbounded γ for near-horizon collisions in the case (10). For the standard BSW effect, with g_ϕ separated from zero, it turned out that it is due to collision between the slow fine-tuned particle and a rapid usual one [8]. But now both particles are usual, no fine-tuning is assumed. Therefore, the explanation of unbounded γ should be different from that given in [8].

A. Basic kinematic formulas

To make presentation self-contained, below we repeat and somewhat enlarge derivation of basic formulas made in [8]. Let us introduce the tetrad basis $h_{(a)\mu}$ that corresponds to the frame of the zero-angular momentum observer (ZAMO) [9]. Then,

$$h_{(0)\mu} = -N(1, 0, 0, 0), \quad (19)$$

$$h_{(1)\mu} = \frac{1}{\sqrt{A}}(0, 1, 0, 0), \quad (20)$$

$$h_{(2)\mu} = \sqrt{g_\theta}(0, 0, 1, 0), \quad (21)$$

$$h_{(3)\mu} = \sqrt{g_\phi}(-\Omega, 0, 0, 1). \quad (22)$$

Here, $x^\mu = (t, r, \theta, \phi)$. If one identifies $h_{(0)\mu}$ with the four-velocity U_μ of such an observer, it is seen from (19) that $U_\phi = 0$, so this does correspond to the ZAMO. The proper time of the observer under discussion can be obtained according to $d\tau_{obs} = -dx^\mu U_\mu$. Then, one can define the tetrad spatial components of the velocity of a particle:

$$V^{(i)} = V_{(i)} = \frac{dx^\mu h_{(i)\mu}}{d\tau_{obs}} = -\frac{u^\mu h_{(i)\mu}}{u^\mu h_{(0)\mu}}. \quad (23)$$

We apply these formulas to the equatorial motion $\theta = \frac{\pi}{2}$ with $A = N^2$. Using (2), (3) and (19) - (22) we obtain

$$-u^\mu h_{(0)\mu} = \frac{X}{mN}. \quad (24)$$

It follows from (4) and (22) that

$$u^\mu h_{(3)\mu} = \frac{L}{m\sqrt{g_\phi}}. \quad (25)$$

Eq. (23) gives us

$$V^{(3)} = \frac{LN}{\sqrt{g_\phi}X} = \frac{bL}{X}. \quad (26)$$

Taking into account (5) and (20) with $A = N^2$, we have the component in the radial direction

$$V^{(1)} = \sigma \sqrt{1 - \frac{N^2}{X^2} \left(m^2 + \frac{L^2}{g_\phi} \right)}. \quad (27)$$

Then, calculating the absolute value $V^2 = (V^{(1)})^2 + (V^{(3)})^2$, we obtain

$$X = \frac{mN}{\sqrt{1 - V^2}}. \quad (28)$$

The gamma factor (8) can be written in the form

$$\gamma = \frac{1}{\sqrt{1 - w^2}}, \quad (29)$$

where w has the meaning of relative velocity. Then,

$$w^2 = 1 - \frac{(1 - V_1^2)(1 - V_2^2)}{(1 - \vec{V}_1 \vec{V}_2)^2} \quad (30)$$

(see, for example, problem 1.3. in [10]).

B. Behavior of a velocity near the horizon

Now, we will use the above general formulas for the analysis of particles' motion near the horizon, when $N \rightarrow 0$. For the angle β between different components of the velocity we have for small N :

$$\tan \beta = -\frac{V^{(3)}}{V^{(1)}} \approx \frac{bL}{X \sqrt{1 - \frac{b^2 L^2}{X^2}}}. \quad (31)$$

Near the horizon, eqs. (27), (28) give us

$$-V^{(1)} = 1 - \frac{L^2 b^2}{2X^2} + O(N^2) \quad (32)$$

$$V \approx 1 - \frac{N^2 m^2}{2X^2}, \quad (33)$$

$$\vec{V}_1 \vec{V}_2 \approx (1 - \frac{L_1^2 b^2}{2X_1^2})(1 - \frac{L_2^2 b^2}{2X_2^2}) + b^2 \frac{L_1 L_2}{X_1 X_2}. \quad (34)$$

Thus $V_1 \rightarrow 1$, $V_2 \rightarrow 1$. Meanwhile, the mutual orientation of particles' velocities depends on b . Now, we will consider different cases separately.

C. $\lim_{N \rightarrow 0} b \neq 0$

We see from (34) that

$$\lim_{N \rightarrow 0} \vec{V}_1 \vec{V}_2 < 1 \quad (35)$$

and it follows from (31) that $\beta \neq 0$. Eq. (30) entails that $w \rightarrow 1$, so $\gamma \rightarrow \infty$. Thus the underlying reason of the effect is that particles approach the horizon under different angles β . This is in sharp contrast with the case of nonzero g_ϕ when any usual particle hits the horizon perpendicularly with the speed approaching that of light that leads to $w < 1$ [8] and finite γ in (29).

In the case under discussion, both particles approach the horizon with the speed of light as well, so both of them are rapid. However, because of vanishing g_ϕ and nonzero b , the projection of, say, velocity of particle 2 onto the direction of motion of particle 1 is less than 1, so this piece of motion is slow. Then, we have collision between a rapid particle and (in this sense) slow one.

There is no need to distinguish case of finite b and $b \rightarrow \infty$ (with $|bL_i| < X_i$ according to (14)). Explanation in question applies to both of them.

D. $\lim_{N \rightarrow 0} b = 0$

It follows from (34) that

$$\vec{V}_1 \vec{V}_2 \approx 1 - \frac{b^2}{2} \left(\frac{L_1}{X_1} - \frac{L_2}{X_2} \right)^2, \quad (36)$$

so the situation is somewhat different. It is seen from (31) that $\beta \rightarrow 0$, any particle hits the horizon perpendicularly as in the standard case [8]. However, there is also a big difference here. It is seen from (30) that whether or not $w \rightarrow 1$ is determined by the competition of

two factors - the rate with which the absolute value of each particle approaches the speed of light and the rate of mutual alignment of particles' velocities. In the standard case, both rates are the same. Indeed, $1 - V^2 = O(N^2)$ and $\vec{V}_1 \vec{V}_2 = 1 - O(N^2)$. As a result, according to (30), w is separated from 1 and the effect of unbounded $E_{c.m.}$ for two usual particles is absent. Meanwhile, in our case, because of the small factor g_ϕ in the expression for b , particles tend to be aligned more slowly. As a result, $w \rightarrow 1$. One can say that although $\beta \rightarrow 0$, some "memory" about small nonzero β (i.e. direction of motion) still remains.

The role of small g_ϕ is displayed if one writes the explicit expression for w near the horizon. It follows from (30), (33) and (36) that

$$w^2 \approx 1 - \frac{4m_1^2 m_2^2 X_1^2 X_2^2}{(L_1 X_2 - L_2 X_1)^4} g_\phi^2. \quad (37)$$

It is seen that just the small factor g_ϕ is responsible for the result $w \rightarrow 1$. If, instead, we put g_ϕ separated from zero, w turns out to be separated from 1, so the effect of unbounded γ and $E_{c.m.}$ is absent.

One reservation is in order. There is an exceptional case when $h \equiv L_1 X_2 - L_2 X_1 = 0$. It follows from (3) that $h = L_1 E_2 - L_2 E_1$ is a constant, so if $h = 0$ on the horizon, it vanishes everywhere. In this case, the main term in the near-horizon expansion of γ having the order $\frac{b^2}{N^2} = \frac{1}{g_\phi}$ vanishes. The next terms contain $\frac{b^4}{N^2} = \frac{N^2}{g_\phi^2}$, so for unbounded γ it is necessary that g_ϕ tend to zero faster than N .

E. Finite nonzero versus vanishing g_ϕ : comparison

It is instructive to summarize the observations made above and compare them to the standard BSW effect due to particle collision near the horizon with $g_\phi \neq 0$. In the standard case, one particle should be critical in the sense that $X_H = 0$ for it [1], [2]. Taking into account definition (3), one obtains $E = \Omega_H L$. Therefore, and one is led to conclusion that rotation is the necessary ingredient for this phenomenon. From another hand, for $g_\phi \rightarrow 0$, the spacetime can be static, rotation is not necessary at all since, according to the explanation above, a small factor g_ϕ does the job. And, both particle are usual now, so $X_H \neq 0$ for each of them.

This can be displayed in Table 1.

	$g_\phi \neq 0$ on horizon	$g_\phi = 0$ on horizon
Rotation	necessary	unnecessary
Particles	one critical and one usual	two usual

Table 1. Comparison of conditions that lead to unbounded $E_{c.m.}$ depending on g_ϕ on the horizon.

VI. EXAMPLES

In this section, we show that several particular models considered before fall in the class under discussion.

A. Stringy black hole

In the dilaton gravity with the electromagnetic field, the following exact solution [11], [12] is known:

$$ds^2 = -\left(1 - \frac{2M}{r}\right)dt^2 + \frac{dr^2}{1 - \frac{2M}{r}} + r(r - r_+)(d\theta^2 + \sin^2\theta d\phi)^2. \quad (38)$$

Here r_+ is the horizon radius,

$$r_+ = \frac{Q^2}{M}e^{-2\varphi_0}, \quad (39)$$

Q is the electric charge, M being mass, φ_0 is the dilaton field at infinity.

Let a black hole be extremal,

$$Q^2 = 2M^2e^{2\varphi_0}. \quad (40)$$

Then,

$$r_+ = 2M, \quad (41)$$

$$\lim_{r \rightarrow r_+} b = \frac{1}{r_+}. \quad (42)$$

In doing so, the horizon area $A = 4\pi \lim_{r \rightarrow r_+} r(r - r_+) = 0$, the surface shrinks to the point.

In [4], collision with participation of charged particles was considered. However, this is not necessary since the effect of unbounded γ persists even for collision of neutral ones. Now, eq. (11) applies and we obtain unbounded γ in agreement with [4].

B. Black hole in Brans-Dicke theory

In the Brans-Dicke theory an exact solution is known (see [6] and references therein):

$$ds^2 = \Delta^{-\frac{2}{2\omega+3}} \sin^{-\frac{4}{2\omega+3}} \theta [-dt^2(1 - \frac{2M}{r}) + r^2 \sin^2 \theta d\phi^2] + \Delta^{\frac{2}{2\omega+3}} \sin^{\frac{4}{2\omega+3}} \theta (\frac{dr^2}{1 - \frac{2M}{r}} + r^2 d\theta^2). \quad (43)$$

Here, ω is the parameter of the Brans-Dicke theory, $\Delta = r(r - 2M)$, the horizon lies at $r_+ = 2M$. This metric represents the Brans-Dicke counterpart of the Schwarzschild metric (denoted for shortness as the BDS metric).

Now,

$$(N^2) = \frac{\Delta^{\frac{2\omega+1}{2\omega+3}}}{r^2}, \quad (44)$$

$$g_\phi = \Delta^{-\frac{2}{2\omega+3}} r^2, \quad (45)$$

where we put $\theta = \frac{\pi}{2}$,

$$b^2 = \frac{\Delta}{r^4} = \frac{r - r_+}{r^3}. \quad (46)$$

Curvature invariants are finite at $r = r_+$ for $-\frac{5}{2} \leq \omega < -\frac{3}{2}$ [6]. In this interval, $2\omega+3 < 0$, and $g_\phi \rightarrow 0$. Simultaneously, $g_\theta \rightarrow \infty$ that compensates the small factor g_ϕ , the horizon area remains finite, $A = 4\pi^2 r_+$. Now, $\lim_{r \rightarrow r_+} b = 0$, so Eq. (13) applies.

In [5], the rotating counterpart of the Kerr metric in the Brans-Dicke theory (so-called the BDK metric) was considered. The result $\gamma \rightarrow \infty$ was obtained. Meanwhile, we see that this effect persists even in the BDS metric, so rotation is not necessary here.

VII. JANIS-NEWMAN-WINICOUR METRIC

This metric [13] - [15] can be written in the form

$$ds^2 = -(1 - \frac{r_+}{r})^\nu dt^2 + \frac{dr^2}{(1 - \frac{r_+}{r})^\nu} + r^2(1 - \frac{r_+}{r})^{1-\nu} (d\theta^2 + d\phi^2 \sin^2 \theta). \quad (47)$$

Now, $N^2 = (1 - \frac{r_+}{r})^\nu$, $g_\phi = r^2(1 - \frac{r_+}{r})^{1-\nu}$, $\lim_{r \rightarrow r_+} g_\phi = 0$, provided $\nu < 1$. Thus for $r = r_+$ the sphere of a constant r shrinks to the point and represents a singular point rather than a regular black hole horizon. In doing so,

$$b^2 = \frac{1}{r^2} (1 - \frac{r_+}{r})^{2\nu-1}. \quad (48)$$

To handle the limit $N \rightarrow 0$, our approach applies. Three different cases can be considered separately. Following [3], we assume that $E_1 = E_2 = m$.

$$1. \quad \nu < \frac{1}{2}$$

In this case, $b \rightarrow \infty$ when $r \rightarrow r_+$. If, additionally, we assume that collision occurs in the common turning point for particles 1 and 2 having angular momenta of different signs we obtain from (7), (18) that

$$E_{c.m.}^2 \approx \frac{4m^2}{(1 - \frac{r_+}{r})^\nu}. \quad (49)$$

This coincides with eq. (29) of [3].

$$\nu = \frac{1}{2}, \quad b(r_+) = \frac{1}{r_+^2} \neq 0$$

Now, we use (7), (11). This corresponds just to eq. (31) of [3].

$$2. \quad \nu > \frac{1}{2}$$

Now, $b(r_+) = 0$, (17) gives us

$$C = \frac{(L_1 - L_2)^2}{2}. \quad (50)$$

Then, we have from (7) and (13) that

$$E_{c.m.}^2 \approx \frac{(L_1 - L_2)^2}{r_+^2 (1 - \frac{r_+}{r})^{1-\nu}} \quad (51)$$

that agrees with eq. (32) of [3] (the extra factor $1/2$ in [3] is an obvious typo in passing from their eq. 31 to eq. 32).

A. Geometry versus material source

In [3], the authors attributed indefinitely high $E_{c.m.}$ to the presence of the scalar field. This was criticized in [5] where the main emphasis was made on the role of interaction between particles and the scalar field that can lead to diminishing $E_{c.m.}$. However, the result of infinite $E_{c.m.}$ for collision of free particles in the BDK metric remained in [5] without explanation. Now we see that it is the key property (10) which is responsible for the effect under discussion. Moreover, the material source that supports such a metric can include other fields than the scalar one. Say, the essential ingredient in the case of metric (38) is the presence of the electromagnetic field along with the scalar one.

VIII. SUMMARY

Thus we developed a general approach to acceleration of particles due to near-horizon collisions in the background of axially symmetric rotating black holes, when $g_\phi = 0$ on the horizon. In the invariant form this can be written as $\eta_\mu \eta^\mu = 0$, where η^μ is the Killing vector responsible for rotation. It turned out that, in contrast to the standard case with $g_\phi \neq 0$, now the effect of unbounded $E_{c.m.}$ is obtained for collision of any two usual (not fine-tuned) particles. The kinematic explanation is found. In contrast to the standard case [8] where a rapid particle hits the slow one near the horizon, now both particles are rapid. The effect is achieved due to the angle between particles.

It is clear from derivation that the source of high $E_{c.m.}$ is not related to the scalar field or any other concrete source. The main factor is the geometry. If condition (10) is satisfied, the effect persists both for naked singularities and regular black hole horizons. If the horizon is regular, vanishing g_ϕ is compensated by high g_θ . In this sense, the effect near regular horizons is connected with their high degree of anisotropy. For naked singularities this is not necessary, the isotropic case is also suitable.

The effect exists both for rotating and static ($\Omega = 0$) metrics. In this sense, rotation is not necessary in contrast to the standard BSW effect. Also, there is no need for the electric charge of particles even for static metrics in contrast to the standard case [16]. The effect under discussion reveals itself even for collision of usual particles, so fine-tuning is not required.

Thus the standard BSW effect and the one considered in the present work give complete unified picture, being its different realizations.

It would be of interest to extend the results of this work further relaxing the condition of axial symmetry.

Acknowledgments

This work was funded by the subsidy allocated to Kazan Federal University for the state assignment in the sphere of scientific activities.

- [1] M. Bañados, J. Silk and S.M. West, Kerr black Holes as particle accelerators to arbitrarily high Energy, Phys. Rev. Lett. **103** (2009) 111102 [arXiv:0909.0169].
- [2] O.B. Zaslavskii, Acceleration of particles as universal property of rotating black holes, Phys. Rev. D **82** (2010) 083004 [arXiv:1007.3678].
- [3] M. Patil and P. Joshi, Acceleration of particles by Janis-Newman-Winicour singularities, Phys. Rev. D **85**, 104014 (2012).
- [4] S. Fernando, String black hole: can it be a particle accelerator ?, Gen. Relativ. Gravit. **46**, 1634 (2014), [arXiv:1311.1455].
- [5] J. Sultana and B. Bose, Particle collisions near a Kerr-like black hole in Brans-Dicke theory, Phys. Rev. D **92**, 104022 (2015).
- [6] H. Kim, New black hole solutions in Brans-Dicke theory of gravity, Phys. Rev. D **60**, 024001 (1999) [arXiv:gr-qc/9811012].
- [7] I. V. Tanatarov and O. B. Zaslavskii, Dirty rotating black holes: regularity conditions on stationary horizons, Phys. Rev. D **86**, 044019 (2012) [arXiv:1206.2580].
- [8] O. B. Zaslavskii, Acceleration of particles by black holes: kinematic explanation, Phys. Rev. D **84**, 024007 (2011) [arXiv:1104.4802].
- [9] J. M. Bardeen, W. H. Press, and S. A. Teukolsky, Rotating black holes: locally nonrotating frames, energy extraction and scalar synchrotron radiation, Astrophys. J. **178**, 347 (1972).
- [10] A. P. Lightman, W. H. Press, R. H. Price, and S. A. Teukolsky, *Problem book in Relativity and Gravitation* (Princeton University Press, Princeton, New Jersey, 1975).
- [11] K. A. Bronnikov and G. N. Shikin, Interacting fields in general relativity theory, Izv. Vuzov SSSR Fiz. **9**, 25 (1977) (Russ. Phys. J. **20**, 1138 (1977)).
- [12] Garfinkle, D., Horowitz, G.T., Strominger, A.: Charged black holes in string theory. Phys. Rev. D **43**, 3140 (1991) Erratum **45** 3888 (1992).
- [13] A. I. Janis, E. T. Newman, and J. Winicour, Reality of the Schwarzschild singularity, Phys.

- Rev. Lett. **20**, 878 (1968).
- [14] M. Wyman, Static spherically symmetric scalar fields in general relativity, Phys. Rev. D **24**, 839 (1981).
 - [15] K. S. Virbhadra, S. Jhingan, and P. S. Joshi, Nature of singularity in Einstein-Massless scalar theory, Int. J. Mod. Phys. D **6**, 357 (1997).
 - [16] O. Zaslavskii, Acceleration of particles by nonrotating charged black holes, Pis'ma ZhETF **92**, 635 (2010) (JETP Letters **92**, 571 (2010)), [arXiv:1007.4598].